Optical vibrons in CdSe dots and dispersion relation of the bulk material

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We examine the dependence on size distribution of the Raman line shape of optical vibron modes in CdSe dots. A detailed comparison with the theory of electron-hole correlated Raman scattering in spherical quantum dots, which includes LO-confined modes, is carried out. The obtained CdSe vibron frequencies are found to map well on the ab initio calculated LO phonon dispersion relations of wurtzite CdSe along the [001] direction of the Brillouin zone. [S0163-1829(98)06808-8]

I. INTRODUCTION

Investigations of the quasi-zero-dimensional heterostructures or quantum dots (QD's) have been carried out in particular for II-VI semiconducting compounds.1 Today, several advanced techniques2 make it possible to grow size selected II-VI quantum dots with spherical shape, normally embedded in a glass matrix. The vibrational modes and the electron optical-vibron interaction of these systems are objects of study due to the interest in the linear and nonlinear optical properties, Raman scattering, and other effects.1 The understanding of the lattice dynamical properties and electron optical vibrational mode interaction of these II-VI low-dimensional heterostructures starts with an adequate knowledge of their bulk phonon dispersion. Unfortunately, the literature is scarce in bulk phonon dispersion data for many of the Cd-VI compounds, the main reason being that inelastic neutron scattering cannot be performed due to the large neutron capture cross section of 113Cd present in natural Cd.3

Raman scattering allows a semiquantitative determination of bulk optical branches by measuring the mode frequencies $\omega_n$ as a function of QD radius. For this purpose, the optical vibrations are considered to be confined to the dots with spherical shape and are assigned an equivalent wave vector $q_n$:

$$ q_n = \frac{\mu_n}{R}, $$

where $\mu_n$ is the $n$th node of the spherical Bessel function $j_n$ and $R$ the QD radius. A similar method has been successfully applied to determine LO and TO bulk phonon dispersion relations along different crystal directions by studying the confined optical phonons in short period quasi-one-dimensional GaAs/AlAs superlattices.4–7

A direct determination of the mode frequencies and the equivalent wavevectors from the Raman spectrum in CdSe QD’s appears to be more complex because of the size distribution of the dots embedded in the matrix. In addition, under normal growth conditions the CdSe QD’s have the wurtzite (hexagonal) structure8 and the crystallographic axes of the nanocrystallites are randomly orientated in the glass matrix. In fact, a detailed comparison with the theory of the Raman line shape in QD semiconductors, including exciton-optical vibron interaction, becomes necessary in order to extract meaningful information on bulk phonon frequencies from the observed Raman spectra of nanocrystallites.

We have recently developed a formalism for calculating first order Raman scattering by spherical semiconductor nanostructures. In this formalism, the Fröhlich interaction with optical vibrational modes is treated within a continuum model which takes into account both the elastic and the dielectric properties of the dot.4,8,9 Neglecting valence band mixing at the Γ point of the Brillouin zone, and in the dipole approximation, it is shown that in the Raman process only excitonic states and vibrational modes with zero angular momentum are allowed.9 Assuming a quadratic negative bulk LO-phonon dispersion the vibron eigenfrequencies are given by

$$ \omega_n^2 = \omega_L^2 - \beta_L \left( \frac{\mu_n}{R} \right)^2, $$

where $\omega_L$ is the bulk LO phonon frequency at the Brillouin zone center and $\beta_L$ a parameter describing the dispersion of the LO phonon in the bulk. Using Eqs. (1) and (2), and the dependence on size distribution of the observed Raman spectra of nanocrystallites we should, in principle, be able to derive the average of the LO-phonons dispersion relation in CdSe.

In this paper we show that the model outlined in Refs. 8,9, together with ab initio phonon dispersion calculations, leads to an understanding of the principal features of the Raman spectra of QD’s in CdSe. We first present a detailed analysis of the Raman line shape in terms of size distribution, phonon linewidths, and photon energy of the exciting laser, whereby...
we stress the role of the excitonic states in resonance with the incoming and scattered light in the observed Raman spectra. We show that the asymmetric broadening observed in the Raman line shape with decreasing mean quantum dot radius is due to the contribution of a distribution of vibron modes selected by the resonance of the exciting laser and the scattered light with the confined excitonic states. Comparison of the theoretical results with the measured dependence of the Raman spectra on the size distribution of the QD brings forth the vibron frequencies as a function of QD radius. We compare them with the corresponding bulk LO-phonon dispersion obtained by \textit{ab initio} lattice dynamical calculations. With this method, and the available samples, it is possible to explore dispersion relations of bulk phonons in CdSe up to 50% into the Brillouin zone.

\section{II. THEORETICAL BACKGROUND}

The average Raman cross section of an ensemble of spherical quantum dots with a size distribution function \( F(R) \) can be expressed as

\[
\left\langle \frac{\partial^2 \sigma}{\partial \Omega \partial \omega_s} \right\rangle = \int F(R) \frac{\partial^2 \sigma}{\partial \Omega \partial \omega_s} (R; \omega_i, \omega_s) dR, \quad (3)
\]

where the cross section of the single QD of radius \( R \) per unit solid angle \( d\Omega \) and per unit scattered light frequency \( d\omega_s \) for incoming light frequency \( \omega_i \) is given by\(^8^,^9\)

\[
\frac{\partial^2 \sigma}{\partial \Omega \partial \omega_s} = S_0 \sum_n \sum_{\mu_1, \mu_2} \frac{\langle F | H_{E-R}^{\dagger} | \mu_2 \rangle \langle \mu_2 | H_{E-V} | \mu_1 \rangle \langle \mu_1 | H_{E-R} | F \rangle}{\left[ \hbar \omega_i - E_{\mu_2}(R) + i \Gamma_{\mu_2} \right] \left[ \hbar \omega_i - E_{\mu_1}(R) + i \Gamma_{\mu_1} \right]} \left( \Gamma / \pi \right)^2 \frac{1}{(\hbar \omega_i - \hbar \omega_s - \hbar \omega_n(R))^2 + \Gamma^2}. \quad (4)
\]
TABLE I. Values of the material parameters used for the numerical calculations. Here $m_e$ and $m_h$ are the electron hole effective mass, respectively, $m_0$ is the bare electron mass, $E_g$ is the bulk band gap and $\varepsilon_0$ the static dielectric constant.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CdSe</th>
</tr>
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<tbody>
<tr>
<td>$E_g$ (eV)</td>
<td>1.865(^a)</td>
</tr>
<tr>
<td>$\omega_0$ (cm(^{-1}))</td>
<td>213.1(^b)</td>
</tr>
<tr>
<td>$m_e/m_0$</td>
<td>0.12(^c)</td>
</tr>
<tr>
<td>$m_h/m_0$</td>
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</tr>
<tr>
<td>$\beta_L$ (ms(^{-1}))</td>
<td>2.969x10(^{-3})(^b)</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>9.53(^d)</td>
</tr>
<tr>
<td>$a$ (nm)</td>
<td>0.42999(^c)</td>
</tr>
<tr>
<td>$c$ (nm)</td>
<td>0.70109(^c)</td>
</tr>
</tbody>
</table>

\(^a\)Reference 15.  
\(^b\)Reference 16.  
\(^c\)Reference 17.  
\(^d\)Reference 18.

The final state $|F\rangle$ of the system consists of single vibrons of frequency $\omega_n(R)$ [statistically created following the distribution function $F(R)$] and a photon of frequency $\omega_0$, $|\mu_1\rangle$ ($i=1$, $2$), refers to the intermediate excitonic states, with energy $E_{\mu_1}(R)$ and lifetime broadening $\Gamma_{\mu_1}$. In Eq. (4) $H_{E,R}^{+}$ ($H_{E,V}$) is the crystal-radiation (exciton-vibron) interaction, $S_0$ is a constant involving physical parameters,\(^7\) and a Lorentzian function is included in order to take into account the vibrion linewidth $\Gamma$. Details of the scattering efficiency calculation considering either free electron-hole pairs or excitons as intermediate states of the QD can be seen elsewhere.\(^8,9\) The resonant character of the Raman process is evidenced by the denominators which appear in Eq. (4). Incoming resonance takes place when the energy of the laser photon coincides with one of the exciton energies $E_{\mu_1}$ while outgoing resonance occurs when the scattered frequency equal $E_{\mu_2}$.\(^10\) Assuming an ensemble of spherical dots with a size distribution, the conditions $\hbar \omega_{\mu_1} = E_{\mu_1}(R)$ or \hbar \omega_{\mu_2} - \hbar \omega_n(R) = E_{\mu_2}(R)$ select the Raman spectra of a set of quantum dots with radii $\{R_i\}$ in either incoming or outgoing resonance. In the following, a precise determination of the resonance dot radii $R_i$ becomes necessary to evaluate the equivalent wave vectors $q_n$ involved in the measured Raman spectra. In our case the energy of the exciton ground state has been fitted as a function of the mean radius to the experimental energy of the $N=1$ exciton state obtained from absorption measurements at 4.2 K using an effective radius (in nm) $R_{\text{eff}} = 3.367 + 0.90785 R$ (the constant term 0.336 should account for possible inaccuracies in the $R$ determination or in the boundary conditions).\(^2\) In Fig. 1(a), we illustrate the allowed exciton energies calculated for the first five $L=0$ levels of CdSe nanocrystals embedded in a glassy matrix. The calculation has been performed as discussed in Ref. 9, using the material parameters given in Table I. The dot radii for certain incoming (dashed lines) and outgoing (dotted lines) resonance energies are shown by pairs of arrows. These “resonant” dots give the main contribution to the Raman line shape, thus selecting those vibrational modes with frequencies $\omega_n(\mu_n,R_i)$ which fulfill a resonance condition. The optical vibrational energies of the first four $l_p=0$

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\(^c\)Reference 17.  
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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Raman cross section as a function of Stokes shift for a 1.8 nm radius CdSe sphere embedded in glass at $\hbar \omega_j = 2.707$ eV and different vibron broadening. (a) $\Gamma = 2$ cm\(^{-1}\), (b) $\Gamma = 4$ cm\(^{-1}\), and (c) $\Gamma = 6$ cm\(^{-1}\). The contributions of $l_p = 0$, $n = 1$, and $l_p = 0$, $n = 2$ vibron modes to the total line shape are displayed by dashed and dotted lines, respectively.}
\end{figure}

modes in a spherical CdSe are shown in Fig. 1(b) as a function of $\Gamma$.

Let us now analyze the Raman spectrum in terms of the size distribution of quantum dots. The relative contribution of the excitonic and vibronic states to the Raman line shape depends on the incoming laser frequency, the excitonic oscillator strength, and the exciton-vibron interaction. According to Fig. 1(b), vibrons with different quantum number $n$ are expected to play a role in the scattering process, especially when the resonance condition selects small radii, and broadening and asymmetry of the Raman spectrum follow from the contribution of several vibron modes evaluated at different radii in the resonance regime. The main contribution to the Raman spectrum comes from QD’s with radii in the range $|R_c-R|<\delta R$, where $R_c$ denotes the radius for which either incoming or outgoing resonance is reached. An estimation of $\delta R$ can be obtained with the following equation:

$$
\Gamma_{\mu} = \frac{\partial E_{\mu}(R)}{\partial R} \delta R.
$$

Equation (5) represents deviations from resonance equal to the exciton lifetime broadening. For $\Gamma_{\mu} = 5$ meV and the
material parameters of Table I a maximum value of \( \delta R \approx 0.03 \text{ nm} \) is obtained with Eq. (5). The frequency uncertainty for the corresponding single vibron \( \delta \omega \) can be cast into the form

\[
\frac{\delta \omega}{\omega} = \left( \frac{\omega}{\omega R_b} \right)^2 \frac{1}{R} - \frac{\delta R}{R}
\]

(6)
giving an inhomogeneous broadening of \( \delta \omega \approx 0.1 \text{ cm}^{-1} \) for \( \delta R = 0.02 \text{ nm}, R = 2 \text{ nm} \), and using the parameters of Table I. Figure 2 shows the calculated Raman spectra for different vibron broadenings \( \Gamma \). For this calculation we have considered an ensemble of CdSe quantum dots described by a Gaussian size distribution function centered at the mean radius \( R_0 = 1.8 \text{ nm} \) and a dispersion (at half maximum) of \( \pm 15\% \). These spectra were obtained for the laser energy \( \hbar \omega_l = 2.707 \text{ eV} \), adding up in Eq. (4) the first 10 excitonic levels with \( L = 0 \) for the optical vibrational modes with \( n = 1, 2, 3, \) and \( 4 \), using the parameters of Table I. A lifetime broadening \( \Gamma_R = 5 \text{ meV} \) was assumed for all excitonic transitions. The spectra for \( \hbar \omega_l = 2.707 \text{ eV} \) shown in Fig. 2 correspond to an incoming resonance with the lowest exciton state for a QD with a radius of 1.85 nm, while the outgoing condition determines a maximum resonance radius of 1.88 nm. The Raman line shapes are almost exclusively due to exciton-vibron interaction with the \( l_p = 0, n = 1 \) and 2 vibron modes, the \( l_p = 0, n = 3 \) and 4 contributions are too small to be seen in the scale of the figures. The spectrum of Fig. 2(a) which corresponds to a vibron linewidth of \( \Gamma = 2 \text{ cm}^{-1} \), shows two well defined peaks related to the \( l_p = 0, n = 1 \) and a small one related to \( l_p = 0, n = 2 \) vibrons. Their relative contributions are determined by the \( n \)-dependent part of the exciton-vibron matrix elements \( \delta \) which drop off rapidly with increasing \( n \). The Raman spectrum for \( \Gamma = 4 \text{ cm}^{-1} \) [Fig. 2(b)] shows, beside the principal structure at 210 cm\(^{-1} \) \( (l_p = 0, n = 1) \), a shoulder at 203 cm\(^{-1} \) due to the \( n = 2 \) vibron. As the intrinsic vibron linewidth increases, the calculated spectrum [Fig. 2(c) for \( \Gamma = 6 \text{ cm}^{-1} \)] exhibits an asymmetric broadening towards the low-frequency side, which results from the negative dispersion of the vibron frequencies with increasing radius [see Fig. 1(b)].

### III. RAMAN LINE SHAPE AND OPTICAL VIBRONS

Figures 3(a)–3(d) show four Raman spectra measured at \( T = 5 \text{ K} \) in CdSe nanocrystals with mean radius \( R_0 = 3.8, 2.6, \)
and 3 l p states. In all analyzed nanostructures the stronger excitonresonance stronger than the outgoing one. The contribution

\[ \omega_{\rm{exc}} \]

with the excitonic states 

\[ \omega_{\rm{exc}} \]

scattering profiles related to outgoing and incoming resonances 

\[ \omega_{\rm{exc}} \]

vibron modes are separated sufficiently in fre-

\[ \nu \]

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\[ \nu \]

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\[ \nu \]

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\[ \nu \]

mean radius

\[ R_0 \]

\[ R_0 \]

in the Brillouin zone by a constant factor to reproduce the

\[ \nu \]

experimental line shapes of different mean size CdSe nanocrystals

\[ \nu \]

with the calculated ones. The curves represent \textit{ab initio} lattice
dynamical calculation, within the mass approximation, of the LO
phonon branches wurtzite CdSe along the [100], [010], and [001] directions of the Brillouin zone (see Ref. 16). Note the excellent
agreement between the experimental data and the dispersion along
[001] (the hexagonal axis).

![Figure 4](image-url)

FIG. 4. A plot of the LO-vibron frequencies of CdSe taken from
the data of Figs. 3 vs the “wave vector” calculated with Eq. (1).
The vibrational frequencies have been obtained by comparing of the
experimental line shapes of different mean size CdSe nanocrystals
with the calculated ones. The curves represent \textit{ab initio} lattice
dynamical calculation, within the mass approximation, of the LO
phonon branches wurtzite CdSe along the [100], [010], and [001] directions of the Brillouin zone (see Ref. 16). Note the excellent
agreement between the experimental data and the dispersion along
[001] (the hexagonal axis).

The somewhat different size of the unit cells of CdS and
CdSe is taken into account by writing the reciprocal lattice
vectors in reduced units (2\pi/c for [001], 2\pi/a for [100] and
[010], where c and a are the parameters of the hexagonal
unit cell) and using the experimental lattice constants given in
Table I. We obtain a LO frequency at the Brillouin zone
center of 235 cm\(^{-1}\) which is \( \sim 10\% \) higher than the experimental
one. We therefore scale all the computed frequencies
in the Brillouin zone by a constant factor to reproduce the
experimental frequency of 213.1 cm\(^{-1}\) at the zone center.

The dispersion computed from the experimental data
reported here according to the procedure described above are

\[ \omega_{\rm{exc}} \]

2.1 and 1.8 nm (open dots). The samples were prepared in a
borosilicate matrix by a diffusion controlled process (details
of the growth procedure are given in Ref. 11). The size
dependence of the spectra in Figs. 3(a)–3(d) involves a
decrease of the optical mode frequency as well as a broadening
of the Raman lines with decreasing QD size. Moreover, the
line shapes show an asymmetrical broadening towards the

\[ \nu \]

The calculated Raman cross sections are represented in
Figs. 3(a)–3(d) by thick solid lines. The spectrum of Fig.
3(a) corresponds to \( h\omega_0 = 2.182 \) eV which is in incoming
resonance with the \( L=0, N=1, 2, \) and 3 excitonic states for
QD radii equal to 3.1, 4.1, and 4.7 nm, respectively [see Fig.
1(a)]. These excitonic states are in outgoing resonance, cal-
culated with the bulk LO phonon frequency \( \omega_{\rm{LO}} \), for nano-
crystal radii \( R_0 = 3.2, 4.3, \) and 4.9 nm, respectively. The scat-
ering profiles related to outgoing and incoming resonances 

\[ \nu \]

non dispersion of bulk CdSe along the main direction of the
Brillouin zone. We have also displayed the LO pho-
on dispersion for each set of “resonant” dots. We have also displayed the LO pho-
oon dispersion for one material into another
which differs from it basically only by a change in atomic
masses. This approximation has been successfully used in
the computation of phonon branches in zinc-blende semi-
conductors with a similar lattice parameter.12 Our force con-
stants were obtained using density functional perturbation
theory13 for the case of wurtzite CdS.3 The dispersion relations
displayed in Fig. 4, were obtained from the force con-
stants of CdS after replacing in the dynamical equation the
mass of S by that of Se. The \textit{ab initio} force constants of CdS
have been checked against inelastic neutron scattering
measurements.9 Such measurement are not yet available for
CdSe, although there are at present in progress, together with \textit{ab initio} phonon dispersion calculations for a \textsuperscript{115}CdSe crys-
tal with the wurtzite structure.

The above procedure provides the possibility to evaluate
vibrational optical frequencies and their equivalent “wave
vectors” \( q_n \) corresponding to the relevant Raman spectra. From the
fits of Figs. 3 the optical frequencies are obtained in terms of
the \( n \)th mode and radius. In Fig. 4 we plot the LO frequen-
cies obtained for CdSe as described above from the scatter-
ing line shape of the four samples of Fig. 3. The correspond-
ing “wave vectors” \( q_n \) are evaluated using Eq. (1) for each
in good agreement with the calculations of the for [001] bulk phonon dispersion obtained with the mass approximation. We do not find any explanation as to why the frequencies measured through the Raman spectra of QD’s embedded in a glass matrix, map so well onto the LO-phonon dispersion along the [001] direction of the Brillouin zone as opposed to the perpendicular directions.

IV. CONCLUSIONS

In summary, we have carried out a detailed analysis of the Raman spectra in CdSe dots with mean radii between 1.8 and 3.8 nm. We have shown that the main contribution to the observed cross section arises from those microcrystallites in the sample whose sizes favor incoming and outgoing resonances. The $n = 1$ and $n = 2$ vibron states yield the main contribution to all studied Raman spectra. The shifts and asymmetric broadenings towards the low-frequency side, observed for perfect nanospheres, should contribute to the broadening of the Raman spectra.

(i) The mixing effects of the hole subbands allows the exciton-vibron interaction with $l_p = 2$ modes including the so-called surfacelike modes. 14

(ii) Dots with nonspherical shapes should relax the $L = 0$ and $l_p = 0$ selection rules, the frequency modes with $l_p \neq 0$, disallowed for perfect nanospheres, should contribute to the broadening of the Raman spectra.

(iii) Defects or impurities in the dots induce Raman scattering with the participation of $l_p = 1$ vibronic modes.

The broad shoulders seen at the low-frequency side of the main peak on the spectra of Figs. 3 can be qualitatively explained by the relaxation of the $l_p = 0$ selection rule. These shoulders are centered around 190 $\text{cm}^{-1}$ and move to lower energies as the mean radius decreases. An evaluation of the $l_p = 1$ modes for CdSe nanospheres reveals a mode at 189.5 $\text{cm}^{-1}$ (the so-called Fröhlich mode). Those resonance radii which pick up frequencies around the Fröhlich frequency select optical vibrations which have a strong electrostatic contribution and, in this way, strong exciton-vibron interaction can be achieved.

Finally, the optical vibron frequencies obtained by the fitting procedure described above can be mapped using the wave vector given in Eq. (1) onto $ab$ initio lattice dynamics calculations for wurtzite CdSe. The obtained frequencies map well onto the LO-phonon dispersion along the [001] direction of the Brillouin zone. Our results are consistent with a negative parabolic dispersion law for the bulk LO phonons, with a phenomenological parameter $\beta_l$ equal to $2.989 \times 10^5$ m/sec, which applies at least over the first half of the bulk Brillouin zone of CdSe.

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