THERMODYNAMIC IDENTITIES

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$C_V = \left(\frac{\partial U}{\partial T}\right)_V; \quad C_P = \left(\frac{\partial H}{\partial T}\right)_P; \quad C_P = C_V + TV\alpha\gamma_V$	Specific heat
$\gamma \equiv V\left(\frac{\partial P}{\partial U}\right)_V = \frac{V}{C_V}\left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial \ln T}{\partial \ln V}\right)_S$	Grüneisen parameter
$\left(\frac{\partial T}{\partial V}\right)_S = -T\left(\frac{\partial P}{\partial T}\right)_V \Big/ \left(\frac{\partial U}{\partial T}\right)_V$	Isochores $\rightarrow$ Isentropes
$\left(\frac{\partial T}{\partial P}\right)_S = \frac{TV\alpha}{C_P} = \frac{T\gamma}{K_S}$	Adiabatic temperature profile
$\alpha \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \gamma\rho C_P/K_T$	Thermal expansion coefficient
$K_T \equiv B \equiv -V\left(\frac{\partial P}{\partial V}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$	Bulk modulus
$K_S \equiv -V\left(\frac{\partial P}{\partial V}\right)_S = (1+\alpha\gamma T)K_T$	Adiabatic bulk modulus
$\gamma_V \equiv \left(\frac{\partial P}{\partial T}\right)_V = \alpha K_T$	Thermal pressure coefficient
$P(V,T) - P(V,T_0) = \int_{T_0}^{T} \alpha(V,T')K_T(V,T')\,dT'$	Thermal pressure
$P(V,T) = -\left(\frac{\partial U}{\partial V}\right)_T + T\left(\frac{\partial P}{\partial T}\right)_V$	Total pressure
$\left(\frac{\partial V}{\partial T}\right)_P = -\left[\left(\frac{\partial U}{\partial P}\right)_T + P\left(\frac{\partial V}{\partial P}\right)_T\right]\frac{1}{T} = \alpha V = \frac{\gamma_V}{K_T}V$	Combined relations
$U = -\left(\frac{\partial(\beta F)}{\partial \beta}\right)_V$	$dU = T\,dS - P\,dV$
$F = U - TS$	$dF = -S\,dT - P\,dV$
$G = U - TS + PV$	$dG = -S\,dT + V\,dP$
$H = U + PV = -T^2\left(\frac{\partial(G/T)}{\partial T}\right)_P \rightarrow \Delta\left(\frac{G}{T}\right)_{P_0} = -\int_{T_0}^{T_1}\frac{H(P_0,T)}{T^2}\,dT$	Gibbs-Helmholtz (thermodynamic integration)
$\beta_2 F_2 - \beta_1 F_1 = \int_{\beta_1}^{\beta_2} U(\beta)d\beta$	Helmholtz Free Energy difference (thermodynamic integration)
$\langle K\rangle = -\frac{1}{2}\left\langle \sum_{j=1}^{N} \mathbf{r}_j \cdot \mathbf{F}_j^{\rm tot}\right\rangle = \frac{3}{2}Nk_BT$	Virial theorem
$P = -\left(\frac{\partial F}{\partial V}\right)_T = -\left(\frac{\partial U}{\partial V}\right)_S = \frac{Nk_BT}{V} + \frac{1}{3V}\left\langle \sum_{i=1}^{N} \mathbf{r}_i \cdot \mathbf{f}_i\right\rangle$	Pressure

MAXWELL RELATIONS

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

OTHER RELATIONS

$$\left(\frac{\partial U}{\partial T}\right)_P = C_P - PV\alpha$$

$$\left(\frac{\partial U}{\partial P}\right)_T = \frac{C_V - C_P}{\alpha K_T} + \frac{PV}{K_T}$$

$$\left(\frac{\partial U}{\partial P}\right)_V = \frac{C_V}{\alpha K_T}$$

$$\left(\frac{\partial U}{\partial V}\right)_P = \frac{C_P}{V\alpha} - P$$

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{C_P - C_V}{V\alpha} - P$$