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Dynamics of two interacting dipoles

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Abstract

We study the deterministic spin dynamic of two interacting magnetic moments with anisotropy and dipolar interaction under the presence of an applied magnetic field, by using the Landau–Lifshitz equation with and without a damping term. Due to different kinds of interactions, different time scales appear: a long time scale associated with the dipolar interaction and a short time scale associated with the Zeeman interaction. We found that the total magnetization is not conserved; furthermore, for the non-dissipative case it is a fluctuating function of time, with a strong dependence on the strength of the dipolar term. In the dissipative case there is a transient time before the total magnetization reaches its constant value. We examine this critical time as a function of the distance between the magnetic moments and the phenomenological damping coefficient, and found that it strongly depends on these control parameters.

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Keywords: Spin dynamics; Dipolar interaction; Non-uniform magnetization field

1. Introduction

The dynamic of interacting magnetic particles which are considered as monodomains is important to model devices for spintronic [1]. The understanding of some magnetic properties, such as hysteresis loops, magnetization reversal, zero field cooling (ZFC), and field cooling (FC) behavior, are fundamental to model a magnetic system to obtain, for example, magnetic susceptibility or magnetoresistance. Therefore, a detailed study of a simple interacting magnetic system is quite important and in order.

The standard approaches to study the dynamics of the magnetization reversal are the Landau–Lifshitz (LL) [2] or the Landau–Lifshitz–Gilbert (LLG) equation [3]. Nonlinear time dependent problems in magnetism have already been studied in many cases, and an account of the state of the art can be found in Ref. [4]. A particular case of the Landau–Lifshitz model, the chaotical behavior of the magnetic moment for an anisotropic magnetic particle in a parametric magnetic field (MF), is reported in Ref. [5]; moreover, the nonlinear aspects of a magnetic particle under circularly polarized field is reported in Ref. [6] and a perturbation technique in order to solve the LLG equation is developed in Ref. [7].

From a theoretical point of view, analytical solutions of the LL or LLG equations are difficult to find due to the nonlinearity of these equations, and only in few cases they have been obtained, as for example, in the problem of a magnetic particle with uniaxial anisotropy in the presence of an applied MF when the easy axis is parallel to the external field [8]. Analytical solutions for different orientations of the easy axis with respect to the MF vector and in

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absence of damping can be found in Refs. [9,10]. When the magnetic system is subjected to short field pulses such that its dynamics is fast enough, then the dissipative effects can be neglected [9]. In this context recently the dynamics of two interacting magnetic nanoparticles was analyzed in the presence of an external magnetic filed, in the absence of damping, in the framework of LL equation [11], and it is found that the enveloped function of the modulus of total magnetization is not a constant.

In the present work, we analyze in detail the deterministic behavior of two interacting anisotropic magnetic particles, via dipole–dipole interaction, in the presence of an applied MF. We consider different regimens of the parameters space in order to understand the corresponding magnetic behaviors. We also analyze the dissipative and non-dissipative situation. We find that the absolute value of the total magnetization is not conserved; furthermore, for the non-dissipative case it is a fluctuating function of time, with a strong dependence on the strength of the dipolar term. In the dissipative case, there is a transient time before the total magnetization reaches its constant value. Also a scaling law for this transient time as a function of the distance between the magnetic moments and the phenomenological damping coefficient is found. The paper is arranged in the following way: in Section 2 the theoretical model is described. In Section 3 the numerical results are discussed. Finally, the conclusions are presented in Section 4.

2. Theoretical model

Let us consider a system of N magnetic particles and assume that each particle can be represented by a magnetic monodomain. The temporal evolution of this system can be modeled by the LL equation and it can be written as

$$\frac{\mathrm{d}\mathbf{m}^i}{\mathrm{d}t} = -\gamma \mathbf{m}^i \times \mathbf{H}^i_{\mathrm{eff}} - \frac{\gamma\lambda}{m^i}\mathbf{m}^i \times (\mathbf{m}^i \times \mathbf{H}^i_{\mathrm{eff}}), \tag{1}$$

where $\mathbf{m}^i$ is an individual magnetic moment with $i = (1, \ldots, N)$, γ is an effective gyromagnetic ratio, and λ is a phenomenological damping coefficient. The corresponding ith effective field, $\mathbf{H}^i_{\mathrm{eff}}$, is given by

$$\mathbf{H}^i_{\mathrm{eff}} = -\nabla_{\mathbf{m}^i}\mathscr{H}, \tag{2}$$

where $\mathscr{H}$ represents the appropriate Hamiltonian for the system. At this point we would like to remark that the structure of Eq. (1) in absence of damping, $\lambda = 0$, has an intrinsic relationship with the Nambu's equation governing the dynamics for a triplet of canonical variables with two motion constants [12]. In case of a single magnetic moment, the triplet of canonical variables is given by $\mathbf{m}$ and the two motion constants are the Hamiltonian and the magnitude of the magnetic moment.

In this work we analyze a model consisting of two interacting magnetic particles, with anisotropy and dipolar interaction in the presence of an external MF. It is interesting to note that in real nano-patterned magnetic media, the magnetic particles have substantial shape anisotropy due to a large aspect ratio of the in-plane particle sizes to its thickness, and in general, the influence of this shape anisotropy is much larger than the effect of inter-particle dipole interaction, and can be even larger than the effect of the external MF. The Hamiltonian describing the model is

$$\mathscr{H} = -\sum_{i=1}^{2}(\mathbf{H}\cdot\mathbf{m}^i + \alpha^i(\mathbf{m}^i\cdot\hat{\mathbf{k}})^2) + d^{-3}\mathbf{m}^1\cdot\mathbf{m}^2 - 3d^{-3}(\mathbf{m}^1\cdot\hat{\mathbf{n}})(\mathbf{m}^2\cdot\hat{\mathbf{n}}), \tag{3}$$

where $\mathbf{H}$ represents the external MF, α^i is the anisotropy constant, $\hat{\mathbf{k}}$ is a unitary vector in the direction of the easy axis, d is the fixed distance between the two magnetic moment, and $\hat{\mathbf{n}}$ is a unitary vector along the direction joining the two particles. Therefore, the two corresponding LL equations given in Eq. (1) with the Hamiltonian (3) are

$$\frac{\mathrm{d}\mathbf{m}^i}{\mathrm{d}t} = -\gamma\mathbf{m}^i \times \left(\mathbf{H} + \frac{\alpha^i}{2}(\mathbf{m}^i\cdot\hat{\mathbf{k}})\hat{\mathbf{k}} - d^{-3}\mathbf{m}^k + 3d^{-3}(\mathbf{m}^k\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}\right) - \frac{\gamma\lambda}{m^i}\mathbf{m}^i \times \left(\mathbf{m}^i \times \left(\mathbf{H} + \frac{\alpha^i}{2}(\mathbf{m}^i\cdot\hat{\mathbf{k}}) - d^{-3}\mathbf{m}^k + 3d^{-3}(\mathbf{m}^k\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}\right)\right), \tag{4}$$

where $(i,k) = (1,2)$ such that $i \neq k$. Note that these equations have a nonlinear coupling due to the interaction and the anisotropy terms. Also, the system presents different time scales depending on the magnitudes of the magnetic interactions, because they are of different nature. In addition, because the individual magnetization magnitudes are constant, the global dynamics of each magnetic moment is reduced to a spherical surface. Thus, with these constraints it is useful to employ spherical coordinates $\{r, \theta, \phi\}$, expressing each magnetic moment by $\mathbf{m}^i = m^i\hat{\mathbf{r}}$. Consequently, system (4) is reduced to four differential equations, which can be written as

$$\dot{\theta}^i = -\gamma\left\{(\lambda H_\theta - H_\phi) + \frac{\alpha^i}{2}m^i k_r(\lambda k_\theta - k_\phi) + 3d^{-3}m^k n_r(n_\phi + \lambda n_\theta)\right\}, \tag{5a}$$

$$\dot{\phi}^i \sin\theta^i = -\gamma\left\{(H_\theta + \lambda H_\phi) + \frac{\alpha^i}{2}m^i k_r(k_\theta + \lambda k_\phi) + 3d^{-3}m^k n_r(n_\theta + \lambda n_\phi)\right\}. \tag{5b}$$

We remark that, in general, the right-hand side of Eqs. (5) are complex functions of the angles, θ and ϕ, and we notice that, in this representation, the dynamics of the individual magnetic moment is almost uncoupled, since the only term that couples them is a constant, which is the magnitude of the magnetization of the other magnetic moment. Therefore, in order to study qualitatively the dynamical behavior of the system, we analyze Eqs. (5); however, because the numerical integration of Eqs. (5) is not trivial due to the angles are not defined in the extremes of their ranges, for full numerical integration we use Cartesian components. In addition, let us note that if the external field is a static field, then this system of differential equations is an autonomous system, and it is possible to use a perturbation technique for obtaining the appropriate qualitative behavior. In the following we will examine the particular case of particles with the same magnitude of their magnetic moments, $m^1 = m^2 = m$, and the same shape anisotropy constant, $\alpha^1 = \alpha^2 = \alpha$.

Notice that due to the dipolar interaction, the total magnetization for two magnetic interacting particles in the absence of damping is not stationary [11]; this condition is also fulfilled in our case. In order to elucidate it, we can write Eq. (4) in the Gilbert form [3]:

$$\frac{\mathrm{d}\mathbf{m}^i}{\mathrm{d}t} = -\gamma_{\mathrm{G}}\mathbf{m}^i \times \left(\mathbf{H} + \frac{\alpha}{2}(\mathbf{m}^i \cdot \hat{\mathbf{k}})\hat{\mathbf{k}} - d^{-3}\mathbf{m}^k + 3d^{-3}(\mathbf{m}^k \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}\right) + \lambda_{\mathrm{G}}\mathbf{m}^i \times \frac{\mathrm{d}\mathbf{m}^i}{\mathrm{d}t}. \tag{6}$$

The LLG equation (6) is implicit with respect to $\mathrm{d}\mathbf{m}^i/\mathrm{d}t$, and it can be transformed in the equivalent normalized Landau–Lifshitz form, where $\gamma_{\mathrm{G}} = \gamma + \lambda$ and $\alpha_{\mathrm{G}} = \lambda/m$. In this way, the dynamic of the total magnetization, $\mathbf{M} = \mathbf{m}^1 + \mathbf{m}^2$, obeys the equation

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = \mathbf{F} + \gamma_{\mathrm{G}}\mathbf{M} \times (\mathbf{H} + \zeta_1(\mathbf{M} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} + \zeta_2\mathbf{M} \times (\mathbf{M} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}) + \lambda_{\mathrm{G}}\mathbf{M} \times \frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t}, \tag{7}$$

where $\zeta_1 = 6d^{-3}$ and $\zeta_2 = 2\alpha$. Eq. (7) describes the behavior of an equivalent single magnetic moment, with bi-axial anisotropy with easy axes $\hat{\mathbf{n}}$ and $\hat{\mathbf{k}}$, in the presence of both the applied MF and an extra fluctuating term, $\mathbf{F} = \mathbf{F}\{\mathbf{m}^1, \mathbf{m}^2, \dot{\mathbf{m}}^1, \dot{\mathbf{m}}^2\}$, which is a function of $\mathbf{m}^1$ and $\mathbf{m}^2$, and their corresponding time derivatives. This extra term can be interpreted as a time dependent fluctuating field by proper change of units. Moreover, from Eq. (7) it is clear that the magnitude of the total magnetization is not conserved but fluctuating in time. Finally, we remark that the dynamic behavior of the total magnetization of the system is totally different whether one considers damping or not. In the next section, we will show this situation.

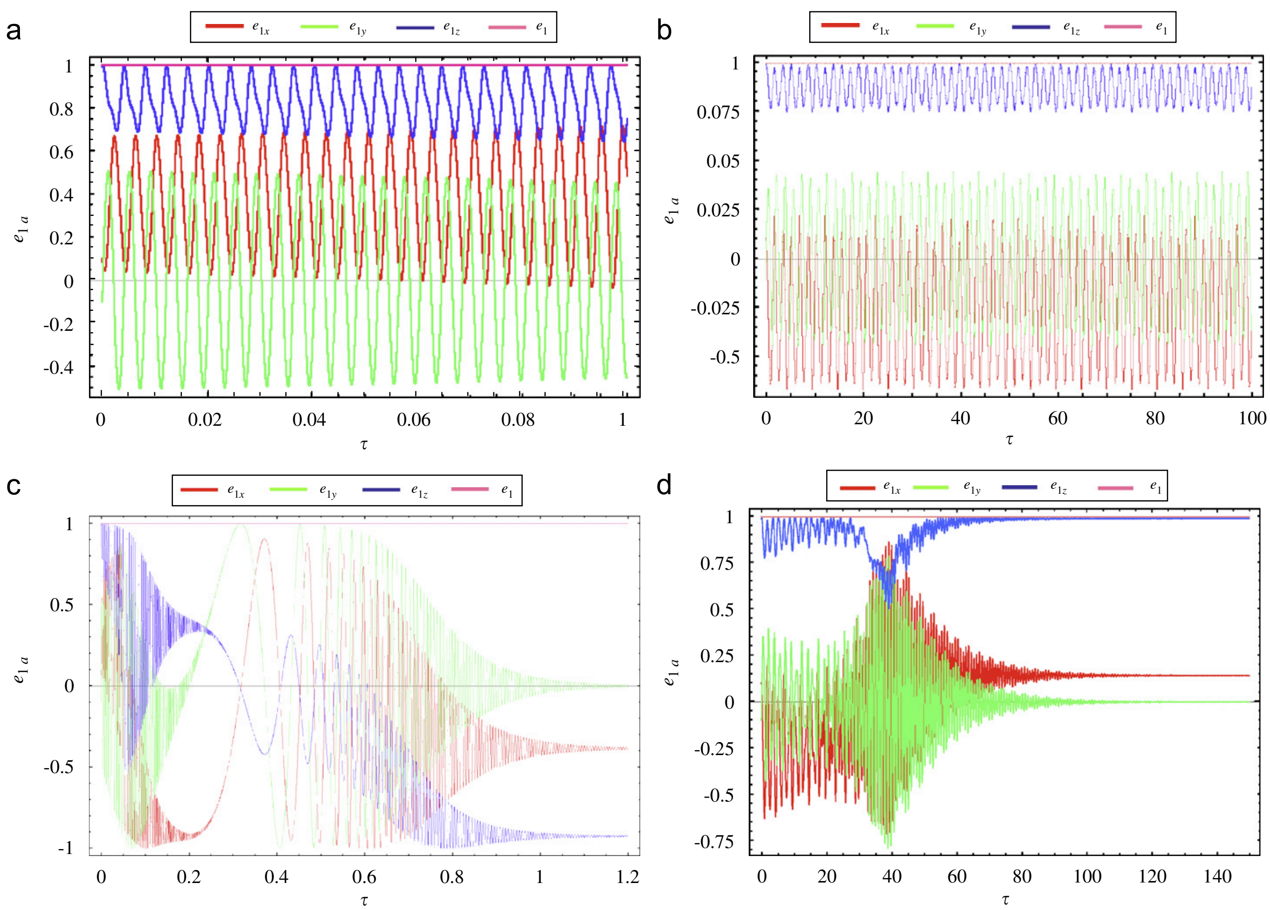

Fig. 1. The magnetization components of $\mathbf{e}^1$ as function of time: (a) for $m/(d^3H_0) = 10^3$ and $\lambda = 0$; (b) for $m/(d^3H_0) = 1$ and $\lambda = 0$; (c) for $m/(d^3H_0) = 10^3$ and $\lambda = 0.01$; and (d) for $m/(d^3H_0) = 1$ and $\lambda = 0.01$. In all figures $\mathbf{h} = 5\hat{\mathbf{z}}$ and $\alpha = 0$.

3. Numerical results

In order to integrate numerically, Eq. (4), we express them in a dimensionless form; for this purpose we introduce the new variables $\mathbf{e}^j = \mathbf{m}^j/m$, $\mathbf{h}^j_{\rm eff} = \mathbf{H}^j_{\rm eff}/H_0$, and $\tau = t/(\gamma H_0)$, where H_0 is the magnitude of a reference MF. Let us define the dimensionless total magnetization system as $\mathbf{E} = \mathbf{e}^1 + \mathbf{e}^2$. An order four Runge–Kutta numerical method was used to solve six differential equations in Cartesian coordinates. The numerical result is arranged as follows: in Sections 3.1 and 3.2 the non-anisotropy and anisotropy case are studied, respectively, for some fixed parameters. In Section 3.3 the non-uniform effect in the modulus of total magnetization is presented. Finally, the qualitative behavior of the system is shown in Section 3.4.

3.1. Non-anisotropic case

In this section, we analyze the case of non-anisotropic case, that is $\alpha = 0$. As the first step, the initial conditions for the magnetic moments are selected as follows: $\mathbf{e}^1(0) = (0.1, -0.1, 0.989949)$ and $\mathbf{e}^2(0) = (-0.1, 0.1, -0.989949)$; notice that these initial conditions are such that the total magnetization vector vanishes. Besides, the fixed parameter is chosen to be $\hat{\mathbf{n}} = (0.382, 0, 0.924)$ and $\mathbf{h} = \mathbf{H}/H_0 = 5\hat{\mathbf{z}}$. Among many possible cases, we show, firstly: (i) when the dipolar field (DF) term is more important than the external MF term and the anisotropy is neglected, and (ii) when the external MF term is more important than the dipolar term and the anisotropy is neglected. The numerical values of the parameters in the DF case is $m/(d^3H_0) = 10^3$, in the MF case is $m/(d^3H_0) = 1$. In the next three figures, the order of the frames are: DF cases are frames (a) and (c) for $\lambda = 0$ and $\lambda = 0.01$, respectively; the MF cases are (b) and (d), for the same λ coefficients.

Fig. 1 shows the time evolution of the three components of $\mathbf{e}^1$. Note that in the non-dissipative case, Figs. 1(a) and (b), the components have a harmonic and quasi-periodic behavior, their initial dephasing being preserved; on the other hand, in the dissipative case, Figs. 1(c) and (d), the dynamical behavior presents two stages, clearly defined: at

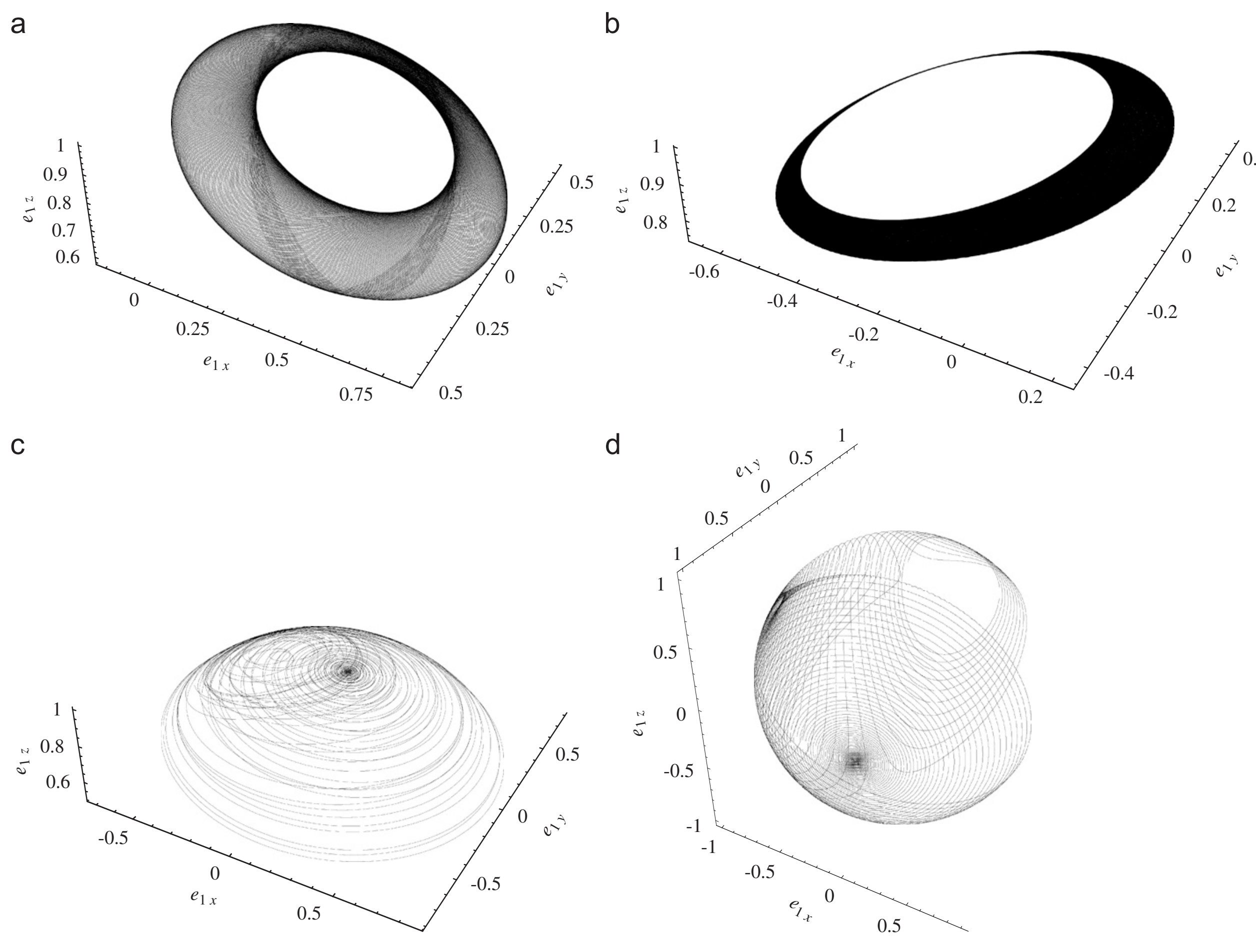

Fig. 2. The 3D phase diagram of components of $\mathbf{e}^1$: (a) for $m/(d^3H_0) = 10^3$ and $\lambda = 0$; (b) for $m/(d^3H_0) = 1$ and $\lambda = 0$; (c) for $m/(d^3H_0) = 10^3$ and $\lambda = 0.01$; and (d) for $m/(d^3H_0) = 1$ and $\lambda = 0.01$. In all figures, $\mathbf{h} = 5\hat{\mathbf{z}}$ and $\alpha = 0$.

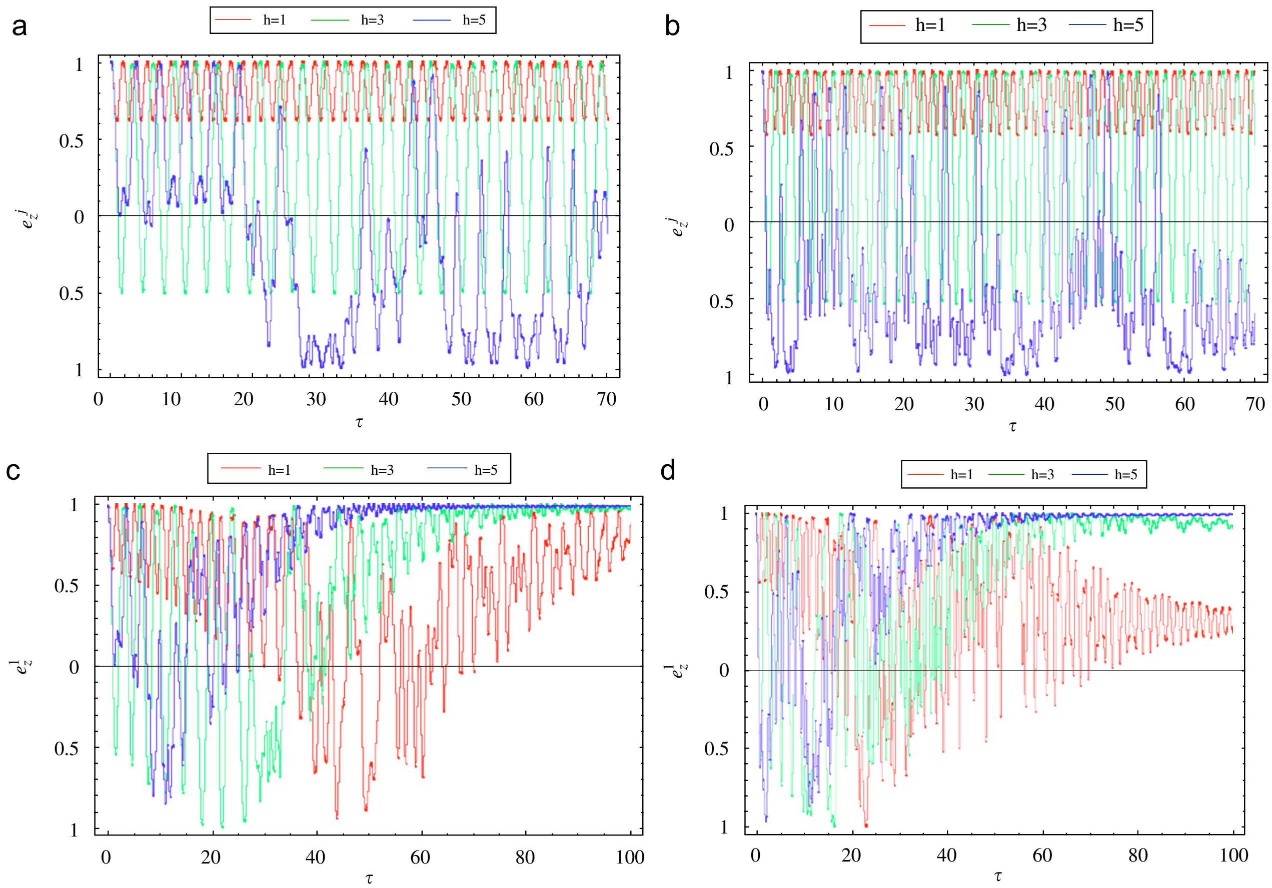

Fig. 3. The z-component of $\mathbf{e}^1$ as function of time with different values of the magnetic field, $\mathbf{h} = (1, 3, 5)\hat{\mathbf{z}}$, at $m/(d^3 H_0) = 1$ for the following situations: (a) $\alpha = 2$ and $\lambda = 0$; (b) $\alpha = 6$ and $\lambda = 0$; (c) for $\alpha = 2$ and $\lambda = 0.01$; and (d) for $\alpha = 6$ and $\lambda = 0.01$.

the beginning of big fluctuations and after that, the individual components take constant values. Also, from the frames of Fig. 1, we observe that, because the two kinds of interactions present in the system, two different time scales appear, the slower one corresponding to DF.

Fig. 2 shows the parametric trajectory of $\mathbf{e}^1$ in the phase space. Note that in the frames (a) and (b) the shape of the surface has holes centered on an axis, which is rotated with respect to z-axis; these holes and the corresponding centers are manifestations of the interaction. In the dissipative case, frames (c) and (d), the shape of the surfaces change roughly; in the DF case the phase diagram is a quasi-sphere with some holes and the MF is a semi-sphere. In both cases there are attractor points.

3.2. Anisotropic case

Now we go through the cases when the anisotropy field is more important than the DF and the external field. In order to analyze the effect of the anisotropy, we suppose that $\hat{\mathbf{k}} = \hat{\mathbf{y}}$. The equations are solved for two different case of dissipation, $\lambda = 0$ and 0.01, and the dipolar parameter $m/(d^3 H_0) = 1$, with the same initial conditions of Section 3.1. Fig. 3 shows z component of the first magnetic moment e_z^1 as function of time for different values of h with two cases of anisotropy constants (a) $\alpha = 2$ and (b) $\alpha = 6$. We note that for conservative (non-dissipative) case (a) and (b) when the MF is smaller than the anisotropy the dynamics is oscillatory and the periods decrease when the anisotropy increases; however, when the MF increases the dynamical behavior of e_z^1 begins to be more complex, in fact, for $h = 5$ it is chaotic. On the other hand, for the dissipative system ($\lambda = 0.01$) the dynamical behavior of e_z^1 changes drastically with respect to the conservative system. Actually, after a certain time interval, e_z^1 goes to a steady value, which increases when the MF grows. From a dynamical system's point of view, this phenomenon is an attractor point. Fig. 4 shows the 3D parametric trajectory of $\mathbf{e}^1$ in the phase space at and for conservative (a) and dissipative dynamics (b), and we note that in the first case there is a chaotic behavior and in the second case the system reaches an attractor.

3.3. Non-uniform magnetization effect

In this section we analyze the effect of dipolar interaction on the evolution of the magnitude of the total

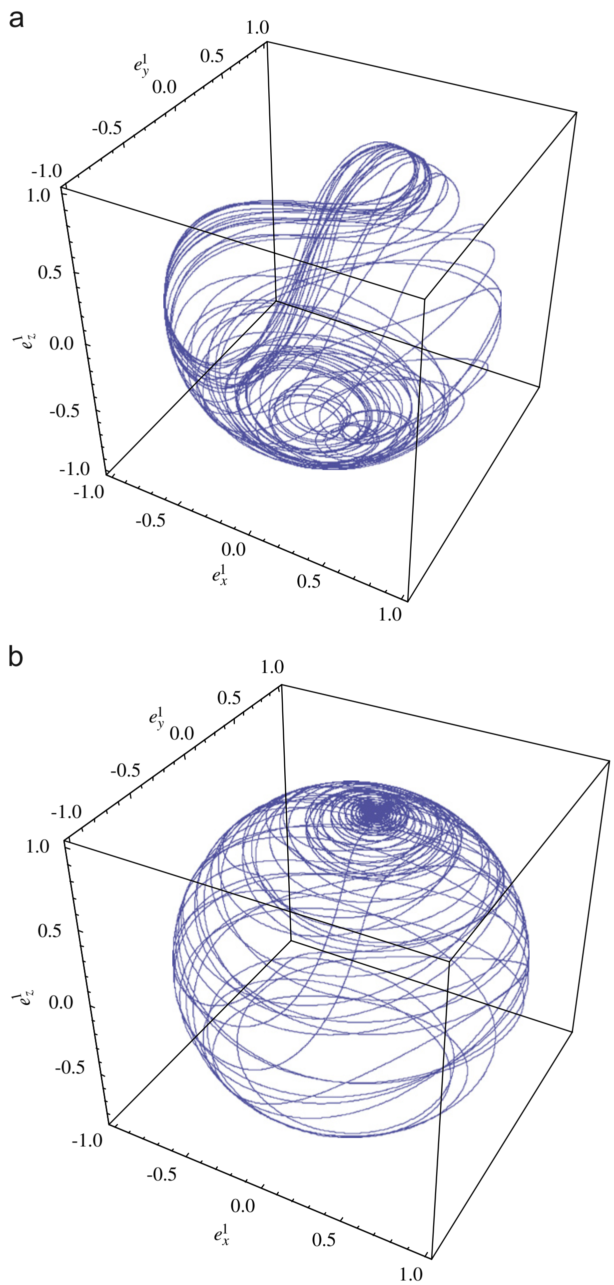

Fig. 4. 3D phase diagram of components of $\mathbf{e}^1$ at $m/(d^3H_0) = 1$, $\mathbf{h} = 5\hat{\mathbf{z}}$ (a) $\alpha = 6$ and $\lambda = 0$; and (b) $\alpha = 6$ and $\lambda = 0.01$.

magnetization. Fig. 5 shows the modulus of total magnetization as function of time for different situations. In particular, Fig. 5(a) is for DF and Fig. 5(b) is for MF when the dissipative effect is not present. It appears clear from these frames that, as a consequence of the dipolar term, the magnitude of the total magnetic moment is not a conserved quantity. In the DF case we found that the magnitude of the total magnetization is a periodic function of time, and in the MF case we detect that it has a quasi-periodic structure. Thus, if we increase the DF we can change the functional form of the total magnetic moment with respect to the time at a fixed MF. Finally, Fig. 5(c) corresponds to DF and Fig. 5(d) corresponds to MF, for $\lambda = 0.01$. In analogy with the non-dissipative case, the total magnetization is not conserved and according to the relative importance of each interaction, the system evolves at different time scales. However, in this case we found that there occur two different stages in the functional form of the magnetization with respect to the time. Firstly, the magnetization is a fluctuating function of the time, and in the final step it remains constant. When the dipolar term is the most important one, we observe that the magnitude of the total magnetization has a local maximum at $\tau = 0.05$, then it decreases and after a local minimum it increases again and takes the highest constant value, which corresponds precisely to the highest value allowed by the theory, that is, the sum of the two individual magnetic moments. In the MF case we observe that the total magnetization magnitude presents rapid fluctuations before reaching the same constant highest value. Notice that the time behavior of the total magnetization strongly depends on the interaction type strengths. The great dissimilarity of the non-dissipative case with respect to the dissipative one is that in the latter the total magnetization attains a constant value after some time, which depends on the interaction strength ratio.

Regarding the dissipative case, it is interesting to analyze in more detail what happens in the transition from the fluctuating regime to the stage where the total magnetization takes its highest stationary constant value. We call critical time τ_c the shortest time at which this transition occur, that is, $E(\tau_c) = E_{st}(\min(\tau))$, E_{st} being the stationary value of E, and it mainly depends on the intermagnetic moments distance d and the damping term λ. Notice that according to Eq. (7), when the anisotropy is not taken into account, the total magnetization can be interpreted as an individual magnetic moment with uniaxial anisotropy, whose strength is proportional to the distance d^{-3} and a fluctuating term. In this way, basically the meaning of the critical time is that at τ_c the fluctuations are not important any longer with respect to the effective anisotropy, ζ_1, and then the total magnetic moment is conserved.

Fig. 6 shows τ_c as a function of λ for different values of $m/(d^3H_0)$ at $\alpha = 0$. We note that in general when λ decreases, the critical time also increases. Moreover, we can observe that $\tau_c(\lambda \to 0) \to \infty$, in agreement with the non-dissipative case, where the dynamical behavior of the total magnetization modulus has an oscillatory behavior [11]. Based on our numerical results, we suggest that the fluctuating function, F, in Eq. (7) has the structure $\mathbf{F} = \boldsymbol{\xi}\Theta(\tau - \tau_c)$, $\Theta(q)$ being the Heaviside distribution and $\boldsymbol{\xi}$ a vector function of the parameters. Finally, from Fig. 7 we note that τ_c is almost steady as a function of the anisotropic constant for short distance.

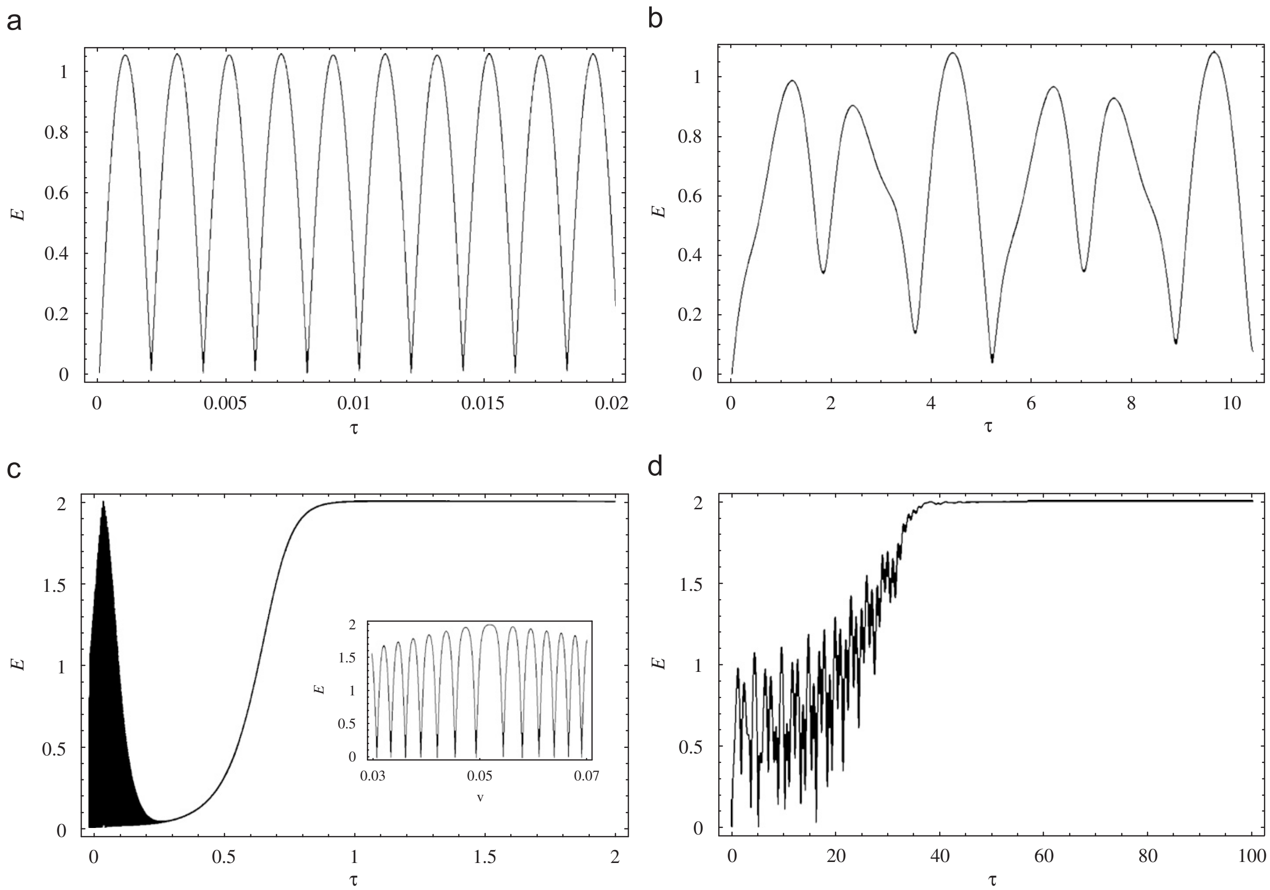

Fig. 5. Magnitude of the total magnetization, E, as function of time: (a) for $m/(d^3H_0) = 10^3$ and $\lambda = 0$; (b) for $m/(d^3H_0) = 1$ and $\lambda = 0$; (c) for $m/(d^3H_0) = 10^3$ and $\lambda = 0.01$; and (d) for $m/(d^3H_0) = 1$ and $\lambda = 0.01$. In all figures $\mathbf{h} = 5\hat{\mathbf{z}}$ and $\alpha = 0$.

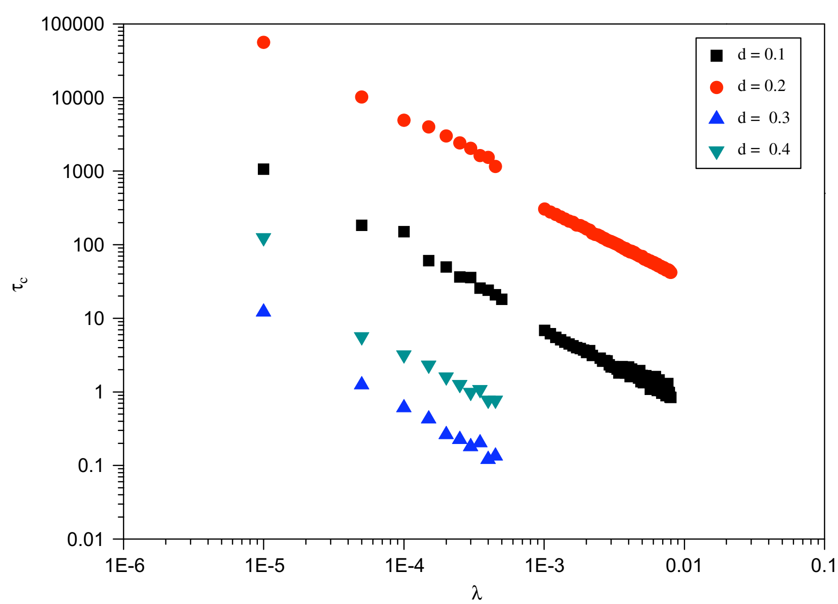

Fig. 6. Critical time τ_c as function of λ for different values of $m/(d^3H_0)$ at $\mathbf{h} = 5\hat{\mathbf{z}}$ and $\alpha = 0$.

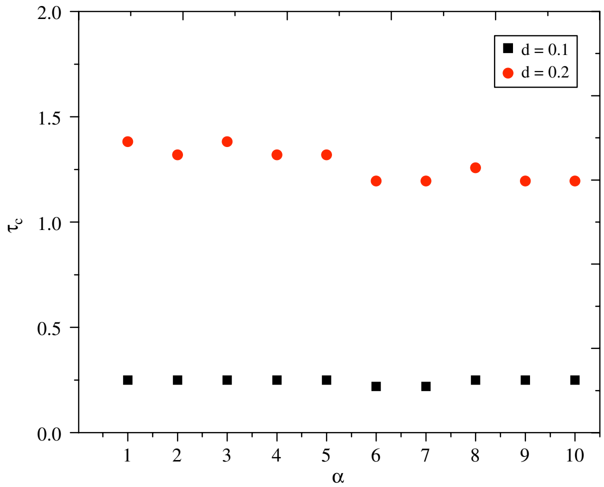

Fig. 7. Critical time τ_c as function of α for different values of $m/(d^3H_0)$ at $\mathbf{h} = 5\hat{\mathbf{z}}$.

3.4. Initial condition explorations

In the preceding sections, we have always used the same initial conditions in order to highlight some important aspects of the different dynamic behavior of the system. In this section we use a set of initial conditions in nearby neighborhoods, to show that the dynamics of the system, for parameters fixed, is robust, i.e., different initial conditions, generally, lead to the same dynamical behavior. It is mainly because the effective field does not depend explicitly on time and we are not in the chaotic regime. Fig. 8 shows the phase diagrams of x–y component of $\mathbf{e}^1$ with different initial conditions for different situations, such that from frame (a) to (c) the dynamics is conservative and from (d) to (e) the dynamics is dissipative. We can observe that in Fig. 8(a) it is shown that the behavior is periodic, and in Figs. 8(b) and (c) the orbits are elliptic, the main difference between them being that with different initial conditions the shape of the ellipses changes. Also, from frame (d) to (e), the system has an attractor point with different transient dynamics.

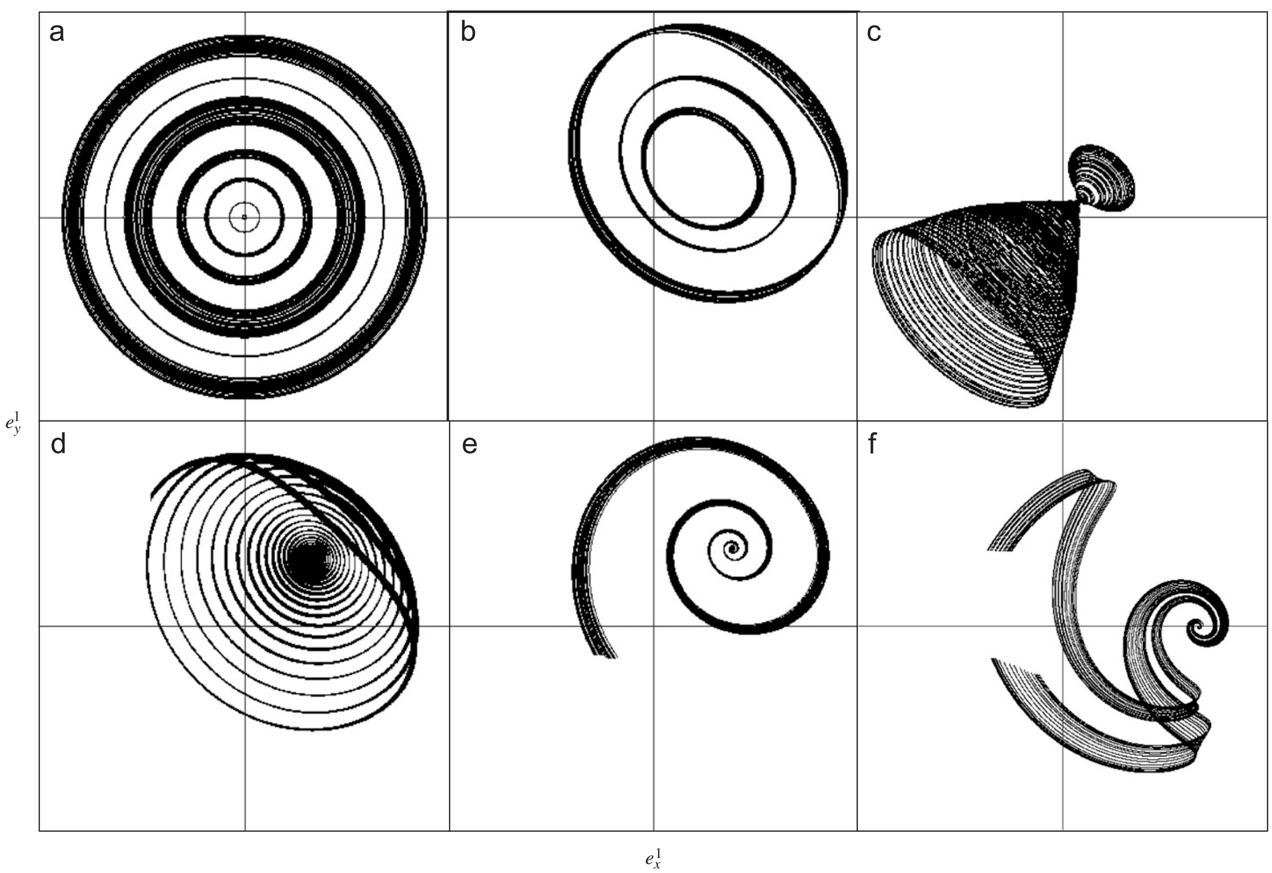

Fig. 8. Phase diagrams of $e_1^x - e_1^y$ for different initial conditions with $m/(d^3H_0) = 10^3$, $\hat{\mathbf{k}} = \hat{\mathbf{z}}$ and $\hat{\mathbf{n}} = \hat{\mathbf{y}}$ at: (a) $\lambda = 0$, $\mathbf{h} = 0.1\hat{\mathbf{z}}$ and $\alpha = 0.1$; (b) $\lambda = 0$, $\mathbf{h} = 0.1(1,1,1)$ and $\alpha = 0.1$; (c) $\lambda = 0$, $\mathbf{h} = 0.1(1,1,1)$ and $\alpha = 0.1$; (d) $\lambda = 0.02$, $\mathbf{h} = 0.1(1,1,1)$ and $\alpha = 0.1$; (e) $\lambda = 0.01$, $\mathbf{h} = 0.1(1,1,1)$ and $\alpha = 0.1$; and (f) $\lambda = 0.2$, $\mathbf{h} = 0.1(1,1,1)$ and $\alpha = 0.1$.

4. Conclusions

The spin dynamic of two interacting magnetic moments using Landau–Lifshitz equation considering anisotropy and dipole–dipole interaction under the presence of an applied MF was analyzed. Due to the two kinds of interactions present in the problem, two time scales arise: a long time scale associated with the dipolar interaction and a short time scale associated with the Zeeman interaction. The dynamical behaviors of non-dissipative and dissipative system are quite different. In the non-dissipative case, the modulus of the total magnetization of the system is always a fluctuating function of time, but in the dissipative case the total magnetic moment has a critical time when it reaches the saturation value; and this time strongly depends on the control parameters of the system. In particular, it increases when the phenomenological damping parameter decreases.

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