Dynamical Behaviour of two Interacting Dipoles

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Abstract. We study the deterministic spin dynamic of two interacting magnetic moments with dipolar interaction under the presence of an applied magnetic field, by using the Landau-Lifshitz equation with damping term. Due to the two kinds of interactions, two time scales appear, a long time scale associated with the dipolar interaction and a short time scale associated with the Zeeman interaction. We found that the total magnetization is not conserved and it strongly depends of the control parameters.

1. Introduction

The dynamic of interacting magnetic particles which are considered as monodomains is important to model devices for spintronic [1]. The understanding of some magnetic properties, such as hysteresis loops, magnetization reversal, zero field cooling (ZFC) and field cooling (FC) behaviour are fundamental to model a magnetic system to obtain, for example, magnetic susceptibility or magnetoresistance; therefore, a detailed study of a simple interacting magnetic system is quite important and in order.

The standard approaches to study the dynamics of the magnetization reversal are the Landau–Lifshitz (LL) [2] or the Landau–Lifshitz–Gilbert (LLG) equation [3]. Nonlinear time dependent problems in magnetism have already been studied in many cases, and an account of the state of the art can be found in Reference [4]. A particular case of the Landau-Lifshitz model, the chaotical behaviour of the magnetic moment for an anisotropic magnetic particle in a parametric magnetic field is reported in reference [5]; moreover, the nonlinear aspects of a magnetic particle under circularly polarized field is reported in reference [6] and a perturbation technique in order to solve the LLG equation, for this problem is developed in the reference [7].

From a theoretical point of view, analytical solutions of the LL or LLG equations are difficult to find due to the nonlinearity of these equations, and only in few cases they have been obtained, as for example in the problem of a magnetic particle with uniaxial anisotropy in the presence of an applied magnetic field when the easy axis is parallel to the external field [8]; analytical solutions for different orientations of the easy axis respect to the magnetic field vector and in absence of damping can be found in references [9–10]. When the magnetic system is subjected to short field pulses such that its dynamics is fast enough, then the dissipative effects can be neglected [9]; in this context recently was analyzed the dynamics of two interacting magnetic nanoparticles in presence of an external magnetic filed, in absence of damping, in the framework of LL equation [11] and it is found that the enveloped function of the modulus of total magnetization is not a constant.

In the present work, we analyze in detail the deterministic behavior of two interacting magnetic particles, via dipole-dipole interaction, in the presence of an applied magnetic field; we consider different regimens of the parameters for understanding corresponding magnetic behaviors. We also analyze the dissipative and nondissipative situation. We find that the absolute value of the total magnetization is not conserved; furthermore, for the non dissipative case it is a fluctuating function of time, with a strong dependence on the strength of the dipolar term. In the dissipative case there is a transient time before the total magnetization reaches its constant value. Also a scaling law for this transient time as a function of the distance between the magnetic moments and the phenomenological damping coefficient is found. The paper is arranged in the following way: In the Sec. 2 the theoretical model is described. In the Sec. 3 the numerical results are discussed. Finally, the conclusions are presented in Sec. 4.

2. Theoretical model

Let us consider a system of *N* magnetic particles and assume that each particle can be represented by a magnetic monodomain. The temporal evolution of this system can be modeled by the LL equation and it can be written as:

$$\dot{\mathbf{m}}^i(t) = -\gamma \mathbf{m}^i(t)\times \mathbf{H}^i_{eff} - \frac{\gamma\lambda}{m^i}\mathbf{m}^i(t)\times\left(\mathbf{m}^i(t)\times\mathbf{H}^i_{eff}\right) \qquad (1)$$

where $\mathbf{m}^i$ is an individual magnetic moment with $i=(1,\ldots,N)$, $\dot{q}$ represent the time dependent derivative of q, γ is an effective gyromagnetic ratio, and λ is a phenomenological damping coefficient. The corresponding *i*-th effective field, $\mathbf{H}^i_{eff}$, is given by $\mathbf{H}^i_{eff} = -\nabla_{\mathbf{m}^i}\mathrm{H}$, where H represents the appropriate Hamiltonian for the system.

In this work we analyze two interacting magnetic particles including dipolar interaction in the presence of an external field; therefore, the Hamiltonian has the following form:

$$\mathrm{H} = -\sum_{i=1}^{2}\mathbf{H}\cdot\mathbf{m}^i + d^{-3}\mathbf{m}^1\cdot\mathbf{m}^2 - 3d^{-3}\left(\mathbf{m}^1\cdot\hat{\mathbf{n}}\right)\left(\mathbf{m}^2\cdot\hat{\mathbf{n}}\right), \qquad (2)$$

where **H** represents the external magnetic field, *d* the fixed distance between the two magnetic moment and **n** is a unitary vector along the direction joining the two particles. We note that corresponding equations have a nonlinear coupling due to the interaction terms and the system presents two time scales depending on the magnitudes of the magnetic interactions, because they are of different nature. Also, because the individual magnetization magnitudes are constant, the global dynamics of each magnetic moment is reduced to a spherical surface. Notice that due to the dipolar interaction, the total magnetization for a two magnetic interacting particles in absence of damping is not stationary [11]; this condition is also fulfilled in our case. In order to elucidate it we can write the equation (4) in the Gilbert form [3]:

$$\dot{\mathbf{m}}^i = -\gamma_G \mathbf{m}^i\times\left(\mathbf{H} - d^{-3}\mathbf{m}^k + 3d^{-3}\left(\mathbf{m}^k\cdot\hat{\mathbf{n}}\right)\hat{\mathbf{n}}\right) + \alpha_G\mathbf{m}^i\times\dot{\mathbf{m}}^i \qquad (3)$$

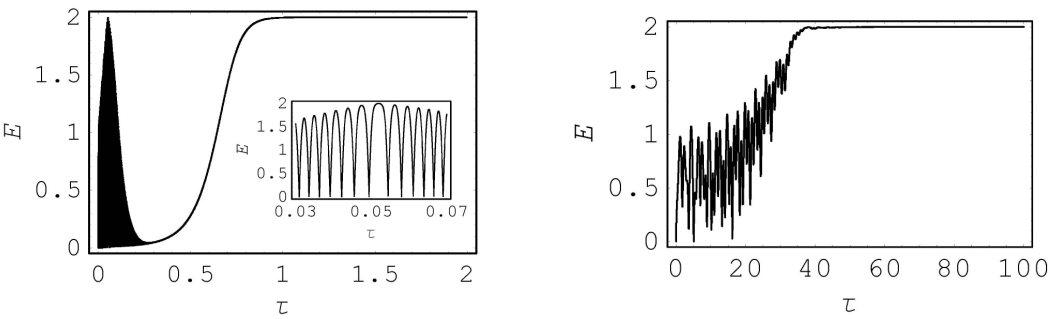
with $(i,k)=(1,2)$ such that $i\neq k$. The LLG equation (3) is implicit with respect to, $\dot{\mathbf{m}}^i$ and it can be transformed in the equivalent normalized Landau-Lifshitz form where $g_G = g + l$ and $a_G = l/m$. Therefore the dynamic of $\mathbf{M} = \mathbf{m}^1 + \mathbf{m}^2$, obeys the equation

$$\dot{\mathbf{M}} = -\gamma_G\mathbf{M}\times\left(\mathbf{H} + 6d^{-3}(\mathbf{M}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}\right) + \alpha_G\mathbf{M}\times\dot{\mathbf{M}} + \mathbf{F}. \qquad (4)$$

The equation (4) describes the behavior of an equivalent single magnetic moment, with uniaxial anisotropy with easy axis $\hat{\mathbf{n}}$, and in the presence of both the applied magnetic field and an extra fluctuating term, $\mathbf{F} = \mathbf{F}\left\{\mathbf{m}^1,\mathbf{m}^2,\dot{\mathbf{m}}^1,\dot{\mathbf{m}}^2\right\}$, which is a function of $\mathbf{m}^1$ and $\mathbf{m}^2$, and their corresponding time derivatives. This extra term can be interpreted as a time dependent fluctuating field by proper change of units. Moreover, from equation (4) it is clear that the magnitude of the total magnetization is not conserved but fluctuating in time. Hence, when the damping coefficient is taking into account, the significant discrepancies are that equation (4) shows a dissipative term involving the total magnetization and that an extra field structure appears, depending on time derivatives of the individual magnetizations. Therefore, the dynamic behavior of the total magnetization of the system considering damping or not will be radically different in each situation. In the next section, we will show this behavior.

3. Numerical Results

In order to integrate numerically the equations (4), we express them in a dimensionless form; for this purpose we introduce the following new variables $\mathbf{e}^j = \mathbf{m}^j/m$, $\mathbf{h}_{eff} = \mathbf{H}_{eff}/H_0$, $\tau = t/(\gamma H_0)$, where H_0 is the magnitude of a reference magnetic field; we define the total dimensionless magnetization of the system by $\mathbf{E} = \mathbf{M}/M$. The initial conditions for the magnetic moments are selected as follows: $\mathbf{e}^1(0) = (0.1, -0.1, 0.989949)$ and $\mathbf{e}^2(0) = (-0.1, 0.1, -0.989949)$; notice that these initial conditions are such that the total magnetization vector vanishes. Besides, the fixed parameters are chosen to be $\mathbf{h} = \mathbf{H}/H_0 = 5\hat{\mathbf{z}}$ and $\hat{\mathbf{n}} = (0.382, 0, 0.924)$. We assume that the two magnetic particles are identical, and use a fourth-order Runge-Kutta method to solve numerically the equations of motion. Among many possible cases, we show two: i) when the dipolar field term (DF) is more important than the external magnetic field term, and ii) when the external magnetic field term (MF) is more important than the dipolar term. The numerical values of the parameters in the DF case is $m/(d^3 H_0) = 10^3$, in the MF case is $m/(d^3 H_0) = 1$.



(a) (b)

Fig. 1: Magnitude of the total magnetization, *E*, as function of time at $\lambda = 0.01$: (a) for $m/(d^3 H_0) = 10^3$; (b) for $m/(d^3 H_0) = 1$.

Figure 1 shows the modulus of total magnetization as function of time for different situations. In particular, Fig. 1 (a) is for DF and Fig. 1 (b) is for MF at $\lambda = 0.01$. In analogy with the nondissipative case, the total magnetization is not conserved and according the relative importance of each interaction, the system evolves at different time scales [11]. However, in this case we found that there occur two different stages in the functional form of the magnetization with respect to the time. Firstly, the magnetization is a fluctuating function of the time, and in the final step it remains constant. However, in this case we found that there occur two different stages in the functional form of the magnetization with respect to the time. Firstly, the magnetization is a fluctuating function of the time, and in the final step it remains constant. When the dipolar term is the most important one, we observe that the magnitude of the total magnetization has a local maximum at $t = 0.05$, then it decreases and after a local minimum it increases again and takes the highest constant value, which corresponds precisely to the highest value allowed by the theory, that is, the sum of the two individual magnetic moments. In the MF case we observe that the total magnetization magnitude present rapid fluctuations before reaching the same constant highest value. Notice that the time behaviour of the total magnetization strongly depends on the interaction type strengths. The great dissimilarity of the nondissipative case with respect to the dissipative case is that in the latter the total magnetization attains a constant value after some time, which depends on the interaction strength ratio.

4. Conclusions

The spin dynamic of two interacting magnetic moments using the Landau-Lifshitz equation considering dipole-dipole interaction under the presence of an applied magnetic field was analyzed. Due to the two kinds of interactions present in the problem, two time scales arise: a long time scale associated with the dipolar interaction and a short time scale associated with the Zeeman interaction. The dynamical behavior in the dissipative case the total magnetic moment has a critical time when it reaches the saturation value; and this time strongly depends of the control parameters of the system.

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